A MODEL OF POPULATION GROWTH INVOLVING MORTALITY-FERTILITY INTERACTIONS: SOME TENTATIVE RESULTS FOR INDIA*

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In analyses of population growth in the low income countries, there have not been many attempts to consider in an integrated and comprehensive way the interactions between the major determinants of change. A reasonably adequate framework of analysis must take into account mortality-fertility interactions in low income societies as significant reductions in mortality are achieved and sustained. Only such an empirically realistic framework of analysis may be expected to provide meaningful insights into the role played by policy and non-policy parameters in the model and to provide a valid basis for making population projections.

There is general consensus that high rates of population growth witnessed in many parts of the low-income world in the last two decades were due, in large measure, to very significant declines in mortality rather than rises in fertility rates. What has remained a controversial issue, however, is the nature and extent of household fertility behaviour response to continuously improving mortality experience.

While still treating mortality changes as exogenous, the present research hypothesizes that birth rates may respond in downward fashion to declines in death rates. The main elements of the hypothesis pertain to household family formation behaviour and are: (a) the concept of Desired Family Size; (b) household response to past mortality changes via lagged adjustment in planned fertility; (c) 'myopic' expectations about future mortality improvements; (d) possible changes in (i) desired family size, (ii) preferred child-spacing pattern and (iii) household behaviour parameters reflecting degree of risk-aversion in response to mortality improvements and the historical consistency of this process. The expectations hypothesis involves distributed lags and myopic expectations. The expected gain in the force of mortality in the future c periods hence, expectations formed at time t is the product of the expected gain in the current period and a myopia factor. Theoretically speaking, the myopia factor may be handled as an Exponential process or a Poisson process (a) independent of or (b) dependent on the past history of the process. The past history of the process will include variations in past mortality gains and the time length of the process. The expectations for the current period involve distributed lags and incorporate the effect of the past history of the process. Mathematically the hypothesis is written as:

$$E\,y\,(t + c/t) = M\,(t + c/t).\ E\,y\,(t/t) \qquad \ldots.(1)$$

where

$$E\,y\,(t/t) = L\,E\,y\,(t - 1/t - 1) + (1 - L)\,y\,(t - 1) \qquad \ldots.(2)$$

where L = lag parameter lying between 0 and 1;

$E\,y\,(t + c/t)$ = expected change in the force of mortality in the period $(t + c)$, expectations formed at time t; and

$y\,(t - 1)$ = actual change in the force of mortality observed in the time period $(t - 1)$.

$M\,(t + c/t)$ = Myopia factor at time t for time period $(t + c)$ in the future.

Changes in mortality rates play an important role in this model on account of the concept of the Desired Completed Family Size and its fixity in the face of changes in mortality. Declining mortality rates and the taking into account of mortality improvements in the decision-making process for determining planned fertility rates imply that under the assumptions made, planned fertility rates respond to changes in mortality via number of currently living children and expected survival rates. Decline in mortality rates will induce declines in planned fertility rates in order to achieve the goal of a fixed DCFS.

In determining planned fertility or the desired number of children to be born over the remaining child-bearing ages of the mother expected survival rates play a critical role. These rates which are subjectively conceived by the family are influenced by the attitude towards past and future mortality changes, attitude toward risk, etc. In many poor traditional societies which are the main focus of this study, where parents may, in a sense, be considered as regarding children as capital on which they depend in their old age, there may be and possibly is a tendency to play "safe" within reasonable limits and these reasonable limits will depend upon family's tastes regarding risk-taking and risk-aversion. For a poor developing society, it may be assumed that people are generally prone to be more of risk-averters type, when the question of their basic livelihood and survival in old age is concerned, recognizing the almost total absence of Social Security programs in these societies. This implies the existence of a bias towards "over-saving" for the future in the form of children. An important relevant issue in this context is that of increasing uncertainties to which parents expose themselves by their dependence on capital in children in a situation of rapidly changing social and cultural values in these societies as economic development occurs. As a consequence, expected income streams in old age from investments in children will be realized with a lower probability than previously. When this holds true, the 'bias' towards over-savings in the form of children will be strengthened. A family's attitude towards risk-taking, 'bias' and other factors will determine the extent to which it will take into account past and expected future mortality improvements to determine its future fertility. These elements are reflected in and captured by the lag parameter L, and the myopia parameter M. Lagged response and myopic expectations are devices that capture these 'biases' and neutralize the reduced probability of realizing expected future income streams.

There may be other important reasons for lagged fertility response to mortality improvements. In realistic situations, households may not be aware of mortality improvements unless they have been underway for quite a while and even then they may not be able to form accurate quantitative judgments about the magnitude of these changes and their impact on family size. Further, in tradition-bound poor societies with a long history of unchanging environment, where traditional values and attitudes have long experience to back them, response to change may be expected to develop only slowly and cautiously. Safe response to these changes may also require the acquisition of knowledge of the use of contraceptives, etc., and of the availability of such devices at reasonable costs. For the present it is not our purpose to make a detailed investigation of the whole host of relevant forces or considerations relevant to low income developing countries that play a part in lagged and partial fertility response by households to declining mortality, but only to visualize the feasibility of such response.

A female population whose family formation behaviour has a goal of achieving a fixed Desired Completed Family Size (DCFS) will respond to mortality changes by appropriate adjustment in their planned fertility. Mortality improvements unaccompanied by any downward adjustments in actual fertility will result in an accelerated population growth of existing numbers and further the households will discover that the number of children surviving to adulthood exceeds the quantity aimed at. Even if instantaneous and 'full' adjustments in planned fertility are made immediately following mortality disturbance and are realized, in the early stages for a time, however, the population will grow at a rate faster than previously on account of the fact that more females would survive to adulthood and higher ages than would have been the case in the absence of any downward disturbances in mortality. Thus mortality improvements will lead in the immediate future to an accelerated rate of population growth even if instantaneous and 'full' fertility adjustments accompany mortality changes. In cases in which fertility responses to mortality declines are neither instantaneous nor 'full' additional sources contributing to accelerated rate of population growth will operate. Both the magnitude and the duration of this process will depend principally upon the lag parameter. In elaborate models in which family formation takes place over a life and time span, the myopia parameter representing expected mortality improvements in the future will also be relevant in determining the sequence of the rates of population growth.

A Simplified Version

A simplified formulation is developed for the purpose of gaining qualitative insights into the role played by model parameters and for throwing into sharp focus the relationship between fertility and mortality rates in determining age composition structure and rate of population growth. The population is divided into four equal age groups 0, 1, 2 and 3. Age group 0 relates to children and age group 1 consists of all adults in child-bearing period of life. Children are born to females in age group 1 only. Since all children are born in one time period, myopia is absent. The myopia parameter $M(t + c/t + c)$ is equal to unity.

Family formation behaviour assumptions are: (i) The family is aiming at a Desired Completed Family Size (DCFS) which is assumed given and fixed and does not change as mortality rates change. DCFS is defined as the number of children born who are desired to survive to adulthood, say age 1. (ii) The family has a fixed preferred child-spacing pattern which does not change as mortality and fertility changes occur. (iii) Families respond to mortality improvements by lagged adjustments in planned fertility. Since a single period covers the whole child-bearing time span, it will be unrealistic to ignore completely mortality changes currently under way whose impact on emerging profile of children living at various ages of the mother's child-bearing span could easily be visible. To take into account the mortality disturbances during the current period, relation (2) has been modified as follows:

$$E\,y(t/t) = L\,E\,y(t - 1/t - 1) + (1 - L)\,y(t) \quad \ldots\ (3)$$

Derivation of Formulae

Let a(x, t+c) denote change in the force of mortality at age x during the time period t+c. The mortality disturbance starts at time t. Prior to time t, age-specific mortality and fertility schedules remained unchanged. Let u(x, t+c) denote the force of mortality at age x at time t+c. When the discussion is general and applies to all age groups, we will, for the sake of brevity, use the notation u(t+c) to refer to the force of mortality at any age at time t+c. We now have:

$$u(t+c+g) = u(t) - a(t) - a(t-1) - \ldots - a(t+c-1) - ga(t+c) \quad \ldots. (4)$$

where a(t+c) refers to improvement in the force of mortality in the relevant age group during time period (t+c), c is an integer and g a fraction.

Let S(x, t-1) refer to before-disturbance one period survival rate schedule. When the discussion is general and applicable to all age groups we will use the notation S(t-1) or simply S as the predisturbance survival rate schedule. Let SR(t+c) refer to actual survival rate for any age during time period (t+c); that is from time (t+c) to (t+c+1), after disturbance. Now

$$u(t+g) = u(t) - ga(t) \quad \ldots. (5)$$

where g lies between 0 and 1. Hence, we have:

$$SR(t) = \exp\left[-\int_0^1 u(t+g)\, dg\right] \quad \ldots. (6)$$

$$= S.\ \exp\left[a(t)/2\right] \quad \ldots. (7)$$

$$SR(t+1) = \exp\left[-\int_0^1 u(t+1+g)\, dg\right]$$

$$= S.\ \exp\left[a(t) + a(t+1)/2\right] \quad \ldots. (8)$$

and so on. In general:

$$SR(t+c) = S. \exp\left[\sum_{c=o}^{c-1} a(t+c) + a(t+c)/2\right] \quad \ldots. (9)$$

Let y(x, t+c) denote the periodic rate of decline in the force of mortality at age x during time period t+c, where y(x, t+c) is a function of x (age) and t+c (time). Let Ey(t+c) denote the expected change in the force of mortality over time period t+c. Using the lag relationship (3) and assuming that no mortality gains were expected prior to time period t, we derive:

$$Ey(t) = (1-L)\ a(t) \quad \ldots. (10)$$

$$Ey(t+1) = (1-L)\ [La(t) + a(t+1)] \quad \ldots. (11)$$

and in general:

$$Ey(t+c) = (1-L)\ [L^c a(t) + L^{c-1} a(t+1) + \ldots a(t+c)] \quad \ldots. (12)$$

Expected survival rates denoted by ESR(t) can be expressed in terms of predisturbance values as follows:

$$ESR(t) = \exp\left[-\int_0^1 Eu(t+g)\, dg\right] \quad \ldots. (13)$$

$$= S.\ \exp\left[(1-L)\ a(t)/2\right] \quad \ldots. (14)$$

Similarly in general:

$$ESR\ (t+c) = S.\ \exp\left[\sum_{c=o}^{c-1} a(t+c).\right] \exp\left[(1-L) \sum_{j=o}^{c} L^{c-j}\ a(t+j)/2\right] \quad \ldots. (15)$$

The households fertility response to mortality improvements is such that the goal of DCFS is to be attained. Let G be the DCFS. If D(t+c) is the planned fertility for period t+c, we have the general relationship:

$$D(t+c).\ ESR(t+c) = D.S. = G.$$

where D is the pre-disturbance total fertility. It can be shown that:

$$D(t) = D.\ \exp\left[-(1-L)\ a(t)/2\right] \quad \ldots. (16)$$

$$D(t+1) = D \exp\left[-a(t)\right].\ \exp\left[-(1-L).\ \{La(t) + a(t+1)\}/2\right] \quad \ldots. (17)$$

$$D(t+c) = D \exp\left[-\sum_{c=o}^{c-1} a(t+c)\right].\ \exp\left[-(1-L) \sum_{j=o}^{c} L^{c-j}.\ a(t+j)/2\right] \quad \ldots. (18)$$

These relations show that the female children born per potential mother will continuously decline from the initial level of D before disturbance and asymptotically approach the full adjustment level of

$$D \exp\left[-\sum_{j=o}^{c-1} a(t+j) - a(t+c)/2\right] \ldots. (19)$$

Let G(t+c) refer to long-term stable population growth factor corresponding to age-specific mortality and fertility schedules of period (t+c). It can be shown that:

$$G(t+c) = D(t+c).\ SR\ (t+c) \quad \ldots. (20)$$

Using expression for SR(t+c) in terms of pre-disturbance values, and substituting also for D(t+c) we have:

$$G(t) = G.\ \exp\ [La(t)/2] \qquad \ldots.(21)$$

$$G(t+1) = G.\ \exp\ [La(t+1)/2 - L(1-L)\ a(t)/2] \qquad \ldots.(22)$$

Similarly, we have, after simplification:

$$G(t+c) = G.\ \exp\ [La(t+c)/2 - L(1-L)\ a(t+c-1)/2 \ \ldots. \ - L^{c}\ (1-L)a(t)/2] \qquad \ldots.(23)$$

Empirical Results for India

For reasons of space, a detailed discussion of the choice of parameter values and of the assumptions underlying the projections is not given here. The following information based on results of 1951, 1961 and 1971 Population Censuses of India is, however, important in making judgments about these assumed values.

(a) The percent growth rates of India's population during 1941-50, 1951-60 and 1961-70 decades were 13.4%, 21.64% and 24.57%. Between 1951 and 1971, India's population increased by 51.1 percent.

(b) If it is assumed that no significant mortality improvements occurred in India in the few decades prior to 1951 so that stable population condition could be taken as a reasonably rough approximation, the long-run stable population one period (20 years) growth factor G may be assumed at $(1.134)^2 = 1.286$. This means that on average, in the absence of significant mortality improvements that actually occurred in India during the fifties and to a much lesser extent during the sixties, India's population between 1951 and 1971 would have increased by 28.6%. The difference of 22.5% may be attributed to mortality and fertility shifts that may have taken place during the 20-year period 1951-71.

(c) Analyses of India's census data suggests that there is little evidence of significant fertility declines occurring during 1951-70 in response to very significant mortality declines underway in that period. This means that the value of lag parameter L in relation (3) is very close to unity.

(d) Based on India's Official Life Tables, the survival rates from birth to age 20 are as follows:

Period	Male	Female
1941-50	.58	.57
1951-60	.72	.71
1961-70	.77	.75

Thus, between 1946 and 1956 (mid-points of the decades), the female's 20-year survival rate increased by 24.56 percent; the percentage for period between 1956 and 1966 was only 5.92 percent. For the 20-year period 1946 to 1966, the 20-year female survival rate increased by 31.93 percent. Evidence is very clear that mortality declines which were very significant during the fifties had considerably slowed down during the sixties. Mortality gains reflected in the above survival rate were of the order of 2.2 percent per year in fifties, but only of 0.6 percent per year in the sixties.

(e) Life expectancy at birth for females was 35 years based on 1941-50 Life Table, 40.0 on 1951-60 Life Table, and 45.6 years on 1961-70 Life Table. Thus, over the 20-years between 1951 and 1971 Censuses, female life expectancy at birth increased by over 10 years, or by nearly 30 percent.

The following assumptions have been made in making population projections:

(i) Calculations have been made for females only. It is assumed that similar orders of magnitude will emerge for males and total population. 50% of children born are assumed female.

(ii) Mortality disturbance is assumed to start at time t that is 1951. It is assumed that $a(t) = .30$; $a(t+1) = .10$, $a(t+2) = .05$ and $a(t+3) = .05$. This means that the forces of mortality between ages 0 and 20 declined on average by amount .30 during 1951-70, will decline by amount .10 during 1971-90 and by amount .05 during 1991-2010 and 2011-2030. In terms of life expectancy, these assumptions are equivalent to assuming that female life expectancy at birth will be 50.0 years in 1980, 55.0 years in 2000, and 57.5 in 2020. Future mortality gains are assumed to be smaller since existing cheap sources of mortality declines are assumed to have been, by and large, almost entirely used up, and further gains are likely to depend on improvements in diet, nutrition, etc., that is, factors which depend on gains in per capita income.

(iii) For making population projections Model Life Tables West-Females for Life Expectancy at Birth equal to 50 years, 55 years and 57.5 years given in Coale and Demeny have been used.

The Summary Table below gives the main results. Projections are made for 2 extreme values of L, the Lag Parameter viz. L=0 representing no lag or full fertility response and L=1 representing no fertility response.

Summary Table

Projections for Female Population for India for Years 1991 and 2010
and Estimation of Important Population Parameters.
(Initial 1971 census figure assumed at 1000)

	Actual 1971	Projected 1991		Projected 2011	
		L=0	L=1.0	L=0	L=1.0
A. Female Population by age-group					
0 (0-20)	506	582	611	718	783
1 (20-40)	286	456	456	537	565
2 (40-60)	148	238	238	393	393
3 (60-80)	60	86	86	147	147
4 Total	1000	1362	1391	1795	1888
B. Proportion age group 0.	.506	.427	.439	.400	.415
C. Proportion age group 1.	.286	.335	.328	.299	.299
D. Proportion "labor force" (=age groups 1 and 2)	.434	.510	.499	.518	.507
E. Dependency Ratio.	.566	.490	.501	.482	.493
F. (i) Estimated total fertility (female children only)	2.53 (1951)	1.78	1.87	1.65	1.70
(ii) Fertility as proportion of 1951 fertility	1	.705	.741	.653	.670
G. Projected Population given actual 1971 total population (millions)	458	746	762	984	1034
H. Rate of Population Increase (%)					
(i) over period	-	36.2%	39.1%	31.8%	35.7%
(ii) annualized rate	-	1.54%	1.65%	1.38%	1.53%
I. Projected Population for year 2001	-	-	-	856	887

Important results are:

(i) If the hypothesis that households' fertility behaviour takes no account of mortality improvements during the current period is true, then India's population is expected to be 762 million by 1991, 887 million by 2001, and 1034 million by 2011.

(ii) India's population increased by 51.6 percent during 1951-71; it is projected to grow by 36.2% under assumption of full fertility response L=0, and by 39.1% under assumption of no fertility response to current mortality improvements during the period 1971-91. The rates of growth for the period 1991-2011 are 31.8% (L=0) and 35.7% (L=1).

(iii) India's child population (age group 0-20) which formed 50.6% in 1971 is projected to fall to 42.7 in 1991 and 40.0% in 2011 under full-fertility response assumption and to 43.9% in 1991 and 41.5% in 2011 under no fertility response assumption.

(iv) Total fertility is expected to fall to 70.5% of its 1951 level by 1991 in full fertility response case (L=0) and to 74.1% in no fertility response case (L=1). These work out to nearly 30 percent and 25 percent declines in total fertility and represent very significant reductions in fertility.

(v) The proportion of female population in child-bearing ages is expected to increase from 28.6% in 1971 to 33.5% if L=0 and 32.8% if L=1 in 1991. After that in the next 20 years 1991-2011 it is expected to fall to about 30 percent on both assumptions.

(vi) The proportion of population in labor force age groups 1 and 2 is expected to rise from 43.4% in 1971 to 51.0% in 1991 and 51.8% in 2010 if full fertility response assumption holds, L=0 case; and to 49.9% in 1991 and 50.7% in 2011 if no fertility response assumption holds, L=1 case.

(vii) The dependency ratio is expected to decline over time on both sets of assumptions.

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APPENDIX 1.1

Value of F(t+c), given a(t)=.30; a(t+1)=.10; a(t+2)=.05 and a(t+3)=.05

F(t+c)	L=0	L=.1	L=.25	L=.5	L=.75	L=1
F(t)	.30	.30	.30	.30	.30	.30
F(t+1)	.10	.130	.175	.250	.325	.40
F(t+2)	.05	.063	.094	.175	.294	.45
F(t+3)	.05	.056	.073	.138	.270	.50

Note: $F(t)=a(t)$

$F(t+1)=La(t)+a(t+1)$

$F(t+c)=La(t+c-1)+a(t+c) \quad c=1,\ 2,\ 3.$

APPENDIX 1.2

Value of J(t+c) where J(t+c) is defined by D(t+c)/D=J(t+c). [Ratio of Total Fertility in any period (t+c) to Total Fertility in the Base Period before disturbance].

J(t+c)	L=0	L=.1	L=.25	L=.5	L=.75	L=1
J(t)	.861	.874	.894	.928	.963	1.00
J(t+1)	.705	.699	.644	.696	.711	.741
J(t+2)	.653	.651	.647	.641	.646	.670
J(t+3)	.622	.622	.621	.616	.617	.638

Note: $J(t+c)=\exp\left[-\sum_{0}^{c-1} a(t+c)\right].$

$\exp\left[-(1-L)F(t+c)/2\right];$

$F(t)=a(t);$

$F(t+c)=L\ a(t+c-1)+a(t+c)$

APPENDIX 1.3

Projection of children born during time periods 1 (1971-1990) and 2 (1991-2010) and surviving to the end of each period.

	Time Period 1 (1971-1990)		Time Period 2 (1991-2010)	
	L=0	L=1.0	L=0	L=1.0
(1)	(2)	(3)	(4)	(5)
1. Female-periods	408	408	514	544
2. Pre-disturbance total fertility	2.53	2.53	2.53	2.53
3. J(t+c)	.705	.741	.653	.670
4. Estimated fertility	1.78	1.87	1.65	1.70
5. Total female births	726	763	848	925
6. Survival Rate	.801	.801	.847	.847
7. Female children living t=1	582	611	718	783

APPENDIX 1.4

Projected 1990 Female Population for India

X	1971	Survival Rate	Survivors 1991
0-1	506	.902	456
1-2	286	.832	238
2-3	148	.579	86

APPENDIX 1.5

Projected 2010 Female Population for India

X		1991	Survival Rate	Survivors 2010
0-1	L=0	582	.924	537
	L=1	611	.924	565
1-2		456	.861	393
2-3		238	.616	147
3-4		86	.0	0

APPENDIX 1.6

Values of exp [K(t+c)] for different values of C and L

C	L=0	L=.1	L=.25	L=.5	L=.75	L=1.0
0	1	1.015	1.038	1.077	1.119	1.161
1	1	.991	.984	.987	.995	1.051
2	1	.997	.990	.981	.982	1.025
3	1	.999	.997	.990	.985	1.025

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